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TABLE OF CONTENTS

TABLE OF CONTENTS

(continued)

ACCESSION for
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I

SUMMARY

This is a progress report on research carried out under the sponsorship of the Air Force Office of Scientific Research under Grant #AF-AFOSR-71-2078D for the period June 1, 1975 to May 31, 1976.

The research accomplished during this period, as well as continuing research, is outlined in Section II. Section III gives the appropriate references, whereas publications resulting from this grant during the reporting period are given in Section IV. Publications by other members of the Center which relate to this research are given in Section V.

The research carried out during this period can be divided into six interrelated areas which arise in the study of control systems.

1) Linear Multivariable Systems
2) Adaptive Control Systems
3) Bilinear Systems
4) Stochastic Systems
5) Systems that give rise to Bifurcations
6) Systems governed by Ordinary and Functional Differential Equations.

These research accomplishments are briefly outlined below; Section II gives a fuller description of these results.

Professor Wolovich and his students have studied the problem of arbitrarily assigning closed loop poles of a linear multivariable

systems developing a new method, a generalization of the classical root locus method. Studies have also been conducted of the attainment of stable solutions of model matching problems. Wolovich has recently published a survey of recent contributions made utilizing the differential operator approach, in contrast to the state-space approach, in the analysis and synthesis of linear multivariable systems.

Professor Pearson and his students have developed a technique, based on a modified minimum energy regulator problem, to obtain feedback stabilization of linear time varying differential systems. Professor Pearson has also developed two methods of parameter identification for linear differential systems.

Using a development in system identification, Pearson has developed identification techniques applicable to a class of parameter adaptive control systems. He has, with a student, studied bilinear control systems with particular applications to parachute gliding systems and the pursuit-evasion missile control problem.

Professors Falb and Wolovich have pursued studies of linear operator feedback for the compensation and control of multivariable systems.

Professor Kushner has developed a number of computational methods and techniques for control problems with diffusion models; these results are presented in a forthcoming monograph. He has also continued his study of the application of Monte Carlo methods for the optimization of constrained noisy systems.

Professor Fleming has recently coauthored a book on determinis-

tic and stochastic optimal control [1]; he has also studied the concept of generalized solutions for optimal stochastic control problems.

The study of bifurcation problems has been pursued by Professor Hale and his students, both from the abstract viewpoint and for specific applications, such as the von Kármán equations for plates and the Duffing equation for nonlinear oscillations.

Professors Banks, Hale and LaSalle have continued their studies of systems described by ordinary and functional differential equations. Professor Hale and students have studied the stability invariance of functional differential equations with respect to changes in the delays. Professor Banks has studied the problem of developing approximation techniques for linear, bilinear and weakly-nonlinear systems with delays. Professor LaSalle has pursued studies of vector liapunov functions and of systems of pure difference equations.

II

RESEARCH ACCOMPLISHMENTS AND CONTINUING RESEARCH

1. Linear Multivariable Systems

a. Analysis and Synthesis of Linear Multivariable Systems

Professor Wolovich and Mr. Panos Antsaklis, one of his graduate students, have been working on the problem of arbitrarily assigning the closed loop poles of a linear multivariable system through the employment of constant gain output feedback. This problem, which is graphically resolved in the scalar case via the classical root locus, remains one of the most important unresolved problems in linear systems theory. Nevertheless, they have succeeded in identifying a real matrix Ω whose rank represents a bound on the maximum number of closed loop poles which can be arbitrarily assigned via constant gain output feedback [2]. Furthermore, examples have been obtained which illustrate that the bound cannot always be attained, and further investigations are planned in order to gain additional insight with respect to this question as well as to develop computational procedures for attaining "as much arbitrary pole placement as possible".

As a result of their investigations, a new method has been found [3] for assigning $\min(n,m+p-1)$ closed loop poles using linear output feedback. Here n is the system order and m and p the number of inputs and outputs respectively. More specifically, parametric expressions of the desired feedback gain matrix H are derived which not only allow the direct assignment of $\min(n,m+p-1)$

closed loop poles, but also make possible the "control" of the remaining unassignable poles. Finally, as a consequence of the above, an interesting generalization of a well known scalar result is presented which constitutes a direct method of assigning min(m,p) closed loop poles.

A partial resolution of the question of stability of solutions to the minimal design problem has also been obtained in terms of transfer matrix factorizations employing the new notions of "common system poles" and "common system zeros" as well as the "fixed poles" of all solutions and those of minimal solutions [4]. It should be noted that the minimal design problem is directly related to the question of designing compensators of lowest possible dynamic order to achieve well-defined closed loop performance. The results obtained are employed to more directly resolve questions involving the attainment of stable solutions to the model matching problem as well as stable minimal order state observers.

Finally, Professor Wolovich has published a rather inclusive report [5] which outlines some of the major recent contributions made utilizing the differential operator approach, rather than the state-space approach, for the analysis and synthesis of linear multivariable systems. It might be noted that a differential operator description of the dynamical behavior of a physical system often follows as a direct result of employing well known physical laws to describe the performance of the system, and techniques which directly utilize this description are often more efficient than those which require the development and employment of equivalent state-space models.

b. Feedback Stabilization of Linear Systems

New results have been obtained in the feedback stabilization of a linear time-varying differential system [6] by Pearson and Kwon. The technique arises from a modified minimum energy regulator problem subject to a terminal constraint on the state. Minimum energy control problems subject to a terminal constraint on the state have been discussed in the literature for various missile control problems and inevitably lead to a singular control law in which the feedback gains are unbounded near the terminal time. Here it is shown that a certain modification of the control law, which avoids the singular property, leads to an asymptotically stable control system. Even when specialized to the time invariant case, the control law leads to an extension of some well-known methods for stabilizing time invariant systems via the inverse of the controllability Gramian matrix. Regarding the latter method for stabilizing discrete time systems, some extensions were obtained this past year which removed the assumption of nonsingularness of the system matrix [7].

c. System Identification

Research in this area by Pearson during the past year has resulted in two methods of parameter identification for linear differential systems which circumvent the need for estimating the system initial conditions when identification utilizes only input-output data observed over a finite time interval $0 \leq t \leq t_1$ of arbitrary duration. In both methods, unknown disturbances are modeled deterministically by uncontrollable modes and the frequencies present in the disturbances, but not the initial conditions exciting such modes,

must be identified along with the system parameters. In the first method, the disturbances are represented implicitly and the frequencies associated with the disturbances must be extracted by a polynomial factorization of the identified transfer function matrix, leaving a reduced order model which represents the controllable portion of the system. A short paper describing this method appeared in [8] and a full-length version, including computer simulation data, will appear in [9]. In the second method, the disturbances are modeled explicitly and the identification procedure involves determining the system and disturbance parameters simultaneously based on input-output data on the time interval $0 \leq t \leq t_1$. The second method, which has been reported in [10], is more general than the first in that the system parameters are allowed to enter nonlinearly into the basic model. The helicopter example in Section 3 of [10] illustrates the importance of this property in that even though the unknown parameters may enter linearly in the state equations, they will nevertheless generally enter nonlinearly when the input-output differential equation is derived. Disturbance parameters always enter nonlinearly with the system parameters in this method due to the manner by which they are modeled, i.e., as uncontrollable modes. Computationally, the first method involves solving linear algebraic (normal) equations for the unknown parameters, followed by a polynomial factorization routine, while the second method leads to scalar valued nonlinear algebraic equations. Computer simulations have not yet been carried out for the second method, but are in the planning stage.

d. Linear Output Feedback Compensation

In order to represent the dynamical behavior of the class of system considered, Falb and Wolovich find it convenient to employ a (general) differential operator representation [11] of the form:

$$P(D)z(t) = Q(D)u(t); \; y(t) = R(D)z(t) + W(D)u(t), \tag{1}$$

where $z(t)$ is a q-vector called the partial state, $u(t)$ is an m-vector called the input, $y(t)$ is a p-vector called the output, and $P(D)$, $Q(D)$, $R(D)$ and $W(D)$ are polynomial matrices of the appropriate dimensions in the differential operator $D = d/dt$ with $P(D)$ $q \times q$ and nonsingular. In certain instances, it will be more useful and illuminating to employ certain specialized forms of (1); i.e., either a controllable differential operator representation ([11]),

$$P_R(D)z(t) = u(t); \; y(t) = R(D)z(t), \tag{1c}$$

or an observable differential operator representation ([11]),

$$P_Q(D)z(t) = Q(D)u(t); \; y(t) = z(t). \tag{1o}$$

It should perhaps be noted that the differential operator representation represents an alternative to (actually a generalization of) a more conventional state-space representation of the form:

$$\dot{x}(t) = Ax(t) + Bu(t); \; y(t) = Cx(t) + Eu(t), \tag{2}$$

where $x(t)$ is an n-vector called the state and A,B,C, and E are real matrices of the appropriate dimensions. In particular,

we note that (2) represents a special form of (1) with $\{DI-A,B,C,D\} = \{P(D),Q(D),R(D),W(D)\}$.

In view of either representation, linear output feedback (lof) is defined by the control law:

$$u(t) = -Hy(t) + v(t) \tag{3}$$

where $H = [h_{ij}]$ is an $m \times p$ real gain matrix and $v(t)$ is an m-dimensional external input. It might be noted that since dynamical elements are not present in (3), lof represents a most practical form of compensation which is frequently employed in the scalar (single input/output) case. The classical root locus, of course, graphically depicts the variation of the poles of a scalar system under lof compensation as a single gain varies over prescribed limits. The implementation simplicity of lof does not, however, imply a corresponding simplicity of analysis in the multivariable case as is well known and documented, due to the nonlinearities introduced by the cross-coupling terms. Nonetheless, numerous investigations ([12], [13], [14], [15], [16]) have been undertaken in order to provide new insight regarding this very practical form of feedback compensation. The most recent and illuminating of these ([13], [14], [15]) have noted that it is "almost always" possible to arbitrarily assign $\min(n,m+p-1)$ closed loop poles via lof. Earlier examples, however, have been given which show that $m + p - 1$ is not generally an upper bound, and very recent studies ([17] have established a new, more illuminating bound on the maximum number of poles which can be arbitrarily assigned via

lof. The results obtained thus far, as well as proposed extensions, will now be delineated.

In particular, if attention is restricted to the case of strictly proper systems, i.e., when $E = 0$ in (2) or, equivalently, when the system transfer matrix:

$$T(s) = C(sI-A)^{-1}B = R(s)P^{-1}(s)Q(s) + W(s) = R(s)P_R^{-1}(s) = P_Q^{-1}(s)Q(s), \quad (4)$$

as derived from (s), (1), (1c), and (1o), respectively, is strictly proper, it follows that the zeros of

$$\Delta_H(s) \triangleq |sI-A+BHC| = |P_R(s)+HR(s)| = |P_Q(s)+Q(s)H| \quad (5)$$

represent the poles of a (state-space or differential operator) system compensated by lof. The dependency of the zeros of $\Delta_H(s)$ on the (pm) gain elements, h_{ij}, of H represents the main focus of this part of the proposal, and a variety of questions related to lof compensation are proposed for investigation, e.g.

(i) How many zeros of $\Delta_H(s)$ can be arbitrarily assigned via H?

(ii) Can a system be stabilized via lof?

(iii) What is the minimum order of a dynamic compensator required to insure complete and arbitrary pole placement?

(iv) What gain matrix, H, or set of matrices assigns certain zeros of $\Delta_H(s)$?

It should be noted that if, as in most earlier investigations, one were to employ a state-space formulation in order to study the variation of the zeros of $|sI-A+BHC|$ as a function of the h_{ij},

then such an investigation would involve the manipulation of more parameters, namely all of the $n(n+m+p)$ entries of $A, B,$ and C, than necessary. A differential operator representation of the form (1c) in comparison, completely describes the dynamical behavior of an equivalent system with no more than $n(m+p) + m^2$ independent terms, a computational savings of at least $n^2 - m^2$ terms. The computational efficiency associated with the differential operator approach manifests itself in many aspects of linear system analysis and synthesis, an observation which will be more thoroughly illustrated in our subsequent discussions.

Let us now be specific regarding the progress made thus far regarding our differential operator investigation of lof compensation and proposed extensions. To begin, we note that in view of (5),

$$\Delta_H(s) = \left| [H \quad I] \begin{bmatrix} R(s) \\ P_R(s) \end{bmatrix} \right|, \tag{6}$$

or, in view of the Binet-Cauchy formula ([18]), $\Delta_H(s)$ can be expressed via the relation:

$$\Delta_H(s) = \sum_{1 \le j_1 \le j_2 \le \cdots j_m \le m+p} [H \quad I] \begin{pmatrix} 1 & 2 & \cdots & m \\ j_1 & j_2 & \cdots & j_m \end{pmatrix} \begin{bmatrix} R(s) \\ P_R(s) \end{bmatrix} \begin{pmatrix} j_1 & j_2 & \cdots & j_m \\ 1 & 2 & \cdots & m \end{pmatrix}, \tag{7}$$

where the notation $G\begin{pmatrix} i_1 & i_2 & \cdots & i_m \\ j_1 & j_2 & \cdots & j_m \end{pmatrix}$ denotes the appropriate m^{th} order minor of G. In other words, in view of (7), $\Delta_H(s) = |P_R(s)+HR(s)|$ can be expressed as the sum of $g \triangleq \binom{m+p}{m} = \binom{m+p}{p}$ products of the

m^{th} order minors of $[H \;\; I]$ and the appropriate m^{th} order minors of $\begin{bmatrix} R(s) \\ P_R(s) \end{bmatrix}$. This, in turn, implies that $\Delta_H(s) - \Delta(s)$, where $\Delta(s) = |P_R(s)| = |sI-A|$, can be expressed as the inner product

$$\Delta_H(s) - \Delta(s) = M_{HI}M_{RP} , \tag{8}$$

where M_{HI} represents a g-1 dimensional row vector consisting of individual and product elements of the h_{ij}, and M_{RP} represents a corresponding column vector consisting of all of the g m^{th} order minors of $\begin{bmatrix} R(s) \\ P_R(s) \end{bmatrix}$ except $\Delta(s) = |P_R(s)|$. We further observe that M_{RP} can be expressed as the product:

$$M_{RP} = \Omega \begin{bmatrix} 1 \\ s \\ \vdots \\ s^{n-1} \end{bmatrix} \tag{9}$$

for some known real $(g-1)xn$ matrix Ω, which can be obtained from either $T(s)$ or (as shown) its factorization, $R(s)P_R^{-1}(s)$. The polynomials which comprise M_{RP} or, equivalently, the matrix defined by (9) play a role in linear system theory which has yet to be fully investigated.[+] To indicate some progress which has been

[+]A portion of the proposed work will address this more general question.

made regarding lof, however, we first define ω as the rank of Ω; i.e.

$$\omega \triangleq \rho[\Omega], \tag{10}$$

and γ as the minimum of ω and mp, where mp represents the number of independent gain elements, h_{ij}, of H: i.e.

$$\gamma \triangleq \min(\omega, mp). \tag{11}$$

In terms of these definitions, the following result can be formally established ([17]).

<u>Theorem 1:</u> <u>No more than</u> γ <u>zeros of</u> $\Delta_H(s)$ <u>can be arbitrarily assigned via</u> H.

It should be noted that γ represents a new and illuminating upper bound on the (maximum) number of poles which can be arbitrarily assigned via lof, one which exceeds $m + p - 1$ (see [13] and [14] in particular) in a large number of cases. This result, of course, does not represent an end in itself but rather a basis for further investigations. In particular, the question of whether or not it is possible to "usually" assign γ zeros of $\Delta_H(s)$ arbitrarily is not resolved by Theorem 1. Further investigations have revealed that in certain cases it is possible while in other cases it is not. To illustrate, if $T(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2+1} \\ \frac{1}{s^2} & \frac{s}{s^2+1} \end{bmatrix} = \begin{bmatrix} s & 1 \\ 1 & s \end{bmatrix} \begin{bmatrix} s^2 & 0 \\ 0 & s^2+1 \end{bmatrix}^{-1} = R(s)P^{-1}(s)$,

then $\Omega = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$, a rank $4(=\omega)$ matrix. In this example, $mp = 4$ as well, so that $\gamma = 4$. Although $\gamma = 4$, it can be shown that it is impossible to come "arbitrarily close" to certain sets of closed loop poles; i.e., if $\Delta_H(s) = \alpha_0 + \alpha_1 s + \cdots + \alpha_3 s^3 + s^4$ then the condition $(\alpha_0+\alpha_2)^2 < 8(\alpha_0+\alpha_1\alpha_3-\alpha_1^2)$ would necessitate the employment of certain complex gain elements h_{ij}. The details associated with this observation will soon appear in [17]. On the other hand, Example 8.2.6 in [11] represents another fourth order, two input, two output systems for which $\gamma = 4$ and complete and arbitrary pole placement via lof is "almost always" possible.

It thus follows, in view of the above, that while the condition $\gamma = n$ is necessary, it is not sufficient to insure complete and arbitrary pole placement for "almost all" sets of closed loop poles. Nevertheless, the approach taken to define Ω and γ is a novel one which has offered, and should continue to provide, significant new insight regarding lof compensation. It is proposed, therefore, that additional investigations be conducted with the eventual goal of obtaining sufficient conditions for "almost always" arbitrarily assigning all n poles of a lof closed loop system when $\gamma = n$. It is felt that certain structural properties of the matrix Ω will play an important role in eventually resolving this, as well as other related questions.

With respect to related questions, it has recently been established that when $\gamma < n$, constraint conditions on the coefficients of $\Delta_H(s)$ can be obtained via (8) independent of the h_{ij}. Investigations are proposed, utilizing these conditions, which will resolve questions related to lof stabilization in such cases as well as those cases when $\gamma = n$, but complete and arbitrary pole placement is not possible (as in the initial example).

Investigations are also underway and proposed regarding the employment of dynamic compensation in combination with lof when $\gamma < n$ but more design flexibility is desired. In particular, it now appears that the observability index associated with the single input/multiple output system with transfer matrix $\frac{M_{RP}}{\Delta(s)}$ will represent a measure of the minimum order of a dynamic compensator required for complete and arbitrary pole placement, although further investigations are required to formalize this observation. To summarize, the eventual goal of all of the research conducted and proposed in this section is to develop practical low order lof compensators for the control of multivariable systems.

2. Adaptive Control

The formulation of the second method for parameter identification described by Pearson in 1c. above has also been shown in Section 4 of [10] to apply to a particular class of parameter adaptive control problems. This class pertains to those feedback control systems in which the unknown plant parameters, w, can be dichotomized into two

sub-vectors, w_a and w_b, i.e., $w = (w_a, w_b)$, in which the vector w_a has a relatively more important affect on the stability of the feedback system than w_b. For example, w_b may contain the parameters for external additive disturbances, modeled as uncontrollable modes, such as wind gust effects, which do not influence the absolute stability of the feedback system, but would lead to erroneous parameter adaptation if ignored in the formulation. The controller portion of the basic feedback control system is assumed to have been structured with sufficient flexibility so that there exists an invertible function Γ between w_a and the controller parameters α, i.e., $\alpha^* = \Gamma(w_a)$, corresponding to which the desired stability and steady state error criteria are upheld uniformly in w_b when $\alpha = \alpha^* = \Gamma(w_a)$. With these basic assumptions, this class of parameter adaptive control problems is shown in [10] to be amenable to the same generic formulation as the second method for parameter identification discussed above. Also, sufficient conditions for the uniqueness of solutions to the nonlinear algebraic equations have been obtained in [10].

3. Control of Bilinear Systems

Various results relating to the control of bilinear systems have emerged in a forthcoming Ph.D. dissertation of Wei [19] under the direction of Pearson. First, it is shown how various nonlinear systems with trigonometric nonlinearities can be re-defined as a bilinear system through a suitable transformation of state variables. Specific examples of such systems are given in relation to a para-

chute gliding system and a pursuit-evasion missile control system. Next, the existence and uniqueness of solutions to a class of minimum energy control problems for commutative bilinear systems is shown resulting from the discovery that the optimal control is a constant vector determined by the boundary conditions. Applications of this result are obtained for the pursuit-evasion missile control problem which falls into the aforementioned class under the assumption that the line speed of the pursuer missile can be controlled in addition to the turn rate. A suboptimal control law is obtained for this class of problems when higher order (actuator) dynamics are included in the model. Simulation studies for the 2 dimensional pursuit-evasion missile control problem have been carried out which include first order actuator dynamics and a least squares estimation algorithm for the target speed and relative heading, in addition to the control algorithm derived for the minimum energy interception problem.

4. Stochastic Control

a. Computational Methods for Control Problems with Diffusion Models

Kushner completed a monograph [20] on the subject, and the preface, describing it in more detail, follows. The monograph deals with a family of interesting and useful techniques for approximating (for computational and other purposes) a large class of optimal stochastic control problems, by simpler optimal stochastic control problems. It also develops a theory and technique for approximating many types of functionals of diffusions that are of interest all through control and communication theory.

This book deals with a number of problems concerning approximations, convergence and numerical methods for stochastic control problems, and also for degenerate elliptic and parabolic equations. The techniques that are developed seem to have a broader applicability in stochastic control theory. In order to illustrate this, in Chapter 11 we give a rather natural approach to the formulation and proof of the separation theorem of stochastic control theory, which is more general than the current approaches in several respects.

The ideas of the book concern a number of interesting techniques for approximating (cost or performance) functionals of diffusions and optimally controlled diffusions, and for approximating the actual diffusion process, defined by stochastic differential equations of the Itô type, both controlled and uncontrolled. Since many of the functionals that we seek to compute or approximate are actually weak solutions of the partial differential equations (i.e., the weak solution can be represented as a functional of an associated diffusion), the techniques for approximating the weak solutions are closely related to the techniques for approximating the diffusions and their functionals. Also, the form of the partial differential equation which is (at least formally) satisfied by a functional of interest, actually suggests numerical methods for the probabilistic or control problem.

We develop numerical methods for optimal stochastic control theory, and prove the required convergence theorems. Neither for

this, nor for any of the other problems, do we require that the cost or optimal cost functions be smooth, or satisfy any particular partial differential equation in any particular sense. Nor do we require, a-priori, that the optimal control exist. Existence is a by product of our method. The numerical techniques are intuitively reasonable, admit of many variations and extensions, and seem to yield good numerical results.

The main mathematical techniques are those related to the use of results in the theory of weak convergence of a sequence of probability measures. The technique seems to provide a point of view which not only suggests numerical methods, but also unites diverse problems in approximation theory and in stochastic control theory. The ideas of weak convergence theory are being used more and more frequently in various areas of applications. But this book, and previous papers by the author and some of his students, seem to be the only currently available works dealing with applications to stochastic control theory or to numerical analysis. The proofs are purely probabilistic. Even when dealing with numerical methods for partial differential equations, we make no explicit smoothness assumptions, and use only probabilistic methods and assumptions.

Chapter I discusses some of the necessary probabilistic background, including such topics as the Wiener process, Markov processes, martingales, stochastic integrals, Itô's Lemma and stochastic differential equations. It is assumed, however, that the reader has some familiarity with the measure theoretic

foundations of probability. In Chapter 2, we describe the basic ideas and results in weak convergence theory, at least in so far as they are needed in the rest of the book.

The computational methods of the book are all equivalent to methods for computing functionals of finite Markov chains, or for computing optimal control policies for control problems with Markov chain models. Many efficient computational techniques are available for these problems. In particular, the functionals for the uncontrolled Markov chains are all solutions to finite linear algebraic equations. The Markov chain can arise roughly as follows. We start with the partial differential equation which, at least, formally, is satisfied by a functional of the diffusion, and apply a particular finite difference approximation to it. If the approximation is chosen carefully (but in a rather natural way), then the finite difference equation is actually the equation that is satisfied by a functional of a particular Markov chain, and we can immediately get the transition probabilities for the chain from the coefficients in the finite difference equation. The local properties of this chain are very close to the local properties of the diffusion, in the sense that there is a natural time scaling with which we interpolate the chain into a continuous parameter process, and the local properties of the interpolation and diffusion are close in certain important respects. Also, the functional of the Markov chain, which is the solution to the approximating equation, is similar in form to a "Riemann sum" approximation to the original functional of the diffusion.

At this point, the theory of weak convergence comes in, and

we show that the functional of the chain does indeed converge to the desired functional of the diffusion, as the difference intervals go to zero. Similarly, the approximation to the weak sense solution to the partial differential equation converges to the weak sense solution. The interpolation of the chain also converges (in a suitable sense) to a solution to the stochastic differential equation. Of course, the finite difference algorithm is classical. But, neither the convergence proofs nor the conditions for convergence are classical. Also, the method can handle a much broader class of functionals than those that may possibly solve some partial differential equation.

It is not necessary that we use finite difference methods; their use does, however, yield an automatic way of generating a family of approximating chains, whether or not the functional is smooth. However, many types of approximations are usable, provided only that they yield the correct limiting properties. Indeed, this versatility is one of the strong points of the approach.

Approximating with Markov chains (whether or not we use classical finite difference techniques) allows us to use our physical intuition - to guide us in the choice of a chain, or in the selection of a computational procedure for solving the equation for the functional of the chain. Our sense of the "dynamics" of the process plays a useful role and can assist us in the selection of procedures which converge faster.

In the case of the optimal control problem, we start by

approximating the non-linear (Bellman) partial differential equation, which is formlly satisfied by the minimal cost function. With a suitable choice of the approximation, the discrete equations are just the dynamic programming equations for the minimal cost function for the optimal control of a certain Markov chain. Again, there are many types of useful approximating chains. This non-linear partial differential equation, or optimal control, case is much more difficult than the uncontrolled or linear partial differential equation case. However, the ideas of weak convergence theory, again, play a very useful role. Under broad conditions, we can show that the sequence of optimal costs for the controlled chain converge to the optimal cost for the controlled diffusion. Indeed, it can even be shown that the (suitably interpolated) chians converge, in a particular sense, to an optimally controlled diffusion.

In Chapter 3, we give the required background concerning the equations satisfied by various functionals of Markov chains, both controlled and uncontrolled. Our method is able to treat optimal control problems with various types of state space constraints. However, this often requires a linear programming (rather than a dynamic programming) formulation, and this is also discussed in Chapter 3.

Chapter 4 discusses the relations between diffusion processes and elliptic and parabolic partial differential equations, both non-degenerate and degenerate and linear and non-linear. Proofs are not given. The representation of the solutions of the linear

equations in terms of path functionals of the diffusion is discussed, as well as the relation between certain non-linear equations and optimal stochastic control problems. Chapter 5 is an introduction to the techniques and results of the sequel. In order to illustrate some of the simpler ideas, the techniques of weak convergence theory are applied to a simple two point boundary value problem for a second order differential equation.

In Chapter 6, we begin the systematic exploitation and development of the ideas. The motivation for the types of approximations is given, and the approximation of a variety of functionals of uncontrolled diffusion and linear elliptic equations is treated. We also show how to approximate an invariant measure of the diffusion, by an invariant measure of an approximating chain, and discuss the use of the approximations for Monte-Carlo, and give some numerical data. The approximations that are explicitly discussed are derived by starting with finite difference techniques; all of them yield Markov chain approximations to the diffusion. However, it should be clear, from the development, that many other methods of approximation can be handled by the same basic techniques. The general approach taken here should motivate and suggest other methods with perhaps preferable properties for specific problems.

Chapter 7 deals with the parabolic equation, and with the probabilistic approach to approximation and convergence for explicit and implicit (and combined) methods. Furthermore, approximations to a (currently much studied) class of non-linear filtering problems are discussed. Some numerical data, concerning approximations to

an invariant measure, is given.

In Chapter 8, we begin the study of non-linear partial differential equations and approximations to optimal control problems, in particular to the optimal stopping and impulsive control problems. The discretizations of the optimization problems for the diffusion yield similar optimization problems on the approximating Markov chains. We are able to prove that the approximations to the optimal processes and cost functions actually converge to the optimal processes and cost functions, resp. The study of non-linear partial differential equations and optimal control problems continues in Chapter 9, where a variety of approximations and control problems are discussed. In order to show that the limiting cost functionals are truly minimal (over some specified class of control policies), and that the limiting processes have the probabilisitc properties of the optimally controlled diffusion, a number of techniques are developed for approximating arbitrary controls, and for proving admissibility or existence. It is expected that many aspects of the general approach will be quite useful in other areas of stochastic control theory. Additional numerical data appears in Chapters 8 and 9. Again, it must be emphasized that much more work needs to be done - to investigate various types of approximations - in order to - fully understand which types of approximations are preferable, and why.

In Chapter 10, we treat two types of extensions of the ideas in Chapters 6 and 7. First, approximations to stochastic differential difference equations, and to path functionals of

such processes, are developed. Then, we discuss the problem of diffusions which are reflected from a boundary, and the corresponding partial differential equations with mixed Neumann and Dirichlet boundary conditions.

Hopefully, the book will help open the door wider to an interesting direction of research in stochastic control theory. Similar techniques can be applied to the problem where the stochastic differential equation has a 'jump term", and the partial differential equations are replaced by partial differential integral equations.

b. <u>Sequential Monte Carlo Methods for Optimizing Constrained Noisy Control Systems</u>

Kushner has continued his investigations [21,22] into the above subject, which has numerous applications in systems optimization. The subject is the Monte Carlo version of nonlinear programming. The results this year were of two types. First, a rather extensive series of computer investigations is underway - concerning the numerical properties of algorithms that were theoretically analyzed last year. Algorithms were of several types, for equality constraints only, Lagrangian methods for inequality
penalty - Lagrangian methods for inequality constraints,
constraints,/and several types of 'pseudo projection' methods. The purpose of the investigation is to gain a thorough understanding of the advantages, shortcomings, numerical properties, etc., of the algorithms - to enable us to improve and develop them. The results, so far, have been extremely good; it appears that the algorithms

are both interesting and useful, and we are well on the way to understanding their numerical properties.

The second type of effort concerned the theoretical properties of the algorithms themselves. Typically, various restrictive conditions were put on the coefficient sequences (such as square summability), and the observation noises were assumed to be uncorrelated. Using some rather powerful ideas in the theory of weak convergence of measures, Kushner has proved the convergence theorems under substantially weaker and more practical conditions.

c. Generalized Solutions in Optimal Stochastic Control

In a paper on generalized solutions in optimal stochastic control [23], Fleming discusses two kinds of such solutions. The first kind is introduced to deal with lack of a Filippov-type convexity condition, much as in ordinary (deterministic) optimal control theory. Results about the existence of an optimum are obtained for stochastic problems in which the data-fields available to the controller do not vary with the control chosen. In particular, these results apply to open loop problems and to problems with completely observed system states. For the latter class of problems, it is noted that the method of dynamic programming frequently gives an ordinary (non-generalized) feedback solution without assuming any convexity conditions.

A difficult open question is the question of existence of optimal controls for stochastic problems with partially observed system states. A second kind of generalized solution is introduced

as a step toward dealing with this matter. Following Benes, Davis-Varaiya, and Bismut, the problem is reformulated as one of finding a Gersanov density whose integral with respect to Wiener measure on the space Ω of possible system trajectories is optimized. In case of partially observed states, the set A of densities corresponding to ordinary controls is neither weakly closed nor convex, in the space $L^2(\Omega)$ of square integrable densities. Generalized controls correspond to points of the weak closure B of the convex hull of A. A partial characterization of points of B is obtained, in terms of auxiliary randomizations.

5. Bifurcation Theory

Hale has continued his work on nonlinear oscillations and bifurcation theory. Chow, Hale and Mallet-Paret [24, 25] have given a general theory of bifurcation for families of mappings which depend on two parameters λ,μ. The complete bifurcation picture is obtained for λ,μ varying independently in a neighborhood of some point. Applications have been given to the von Kármán equations for a rectangular plate and thin shells with lateral loading and normal loading.

In his thesis directed by Hale, List [26] considers the above parameters as well as an additional one concerned with the shape of the plate. Other applications are contained in Hale [27, 28].

Hale and Rodrigues [29] have been discussing the classical forced Duffing equation with and without damping and have been

attempting to characterize the behavior of the periodic solutions as a function of the parameters and allowing the parameters to vary independently. Surprisingly, no one has given the bifurcation diagram for this simple equation. The discussion requires an extension of the methods previously mentioned above.

6. Control of Systems Governed by Ordinary and Functional Differential Equations

a. Functional Differential Equations: Stability and Periodic Solutions

Hale has continued to develop the general theory of functional differential equations both of retarded and neutral type. In the area of stability, he has given a rather complete description of the behavior near a constant solution [30]. This theory gives a description of the center manifold theorem as well as practical methods of determining stability in critical cases. The Hopf bifurcation theorem for ordinary differential equations can also be generalized by using these results. In the development of this theory, a special transformation was devised which permits one to obtain a vector field on the center manifold. This transformation has proved to be very useful in the study of combined sets of differential-difference and difference equations which occur often in the theory of gas dynamics and transmission lines (see [31]).

For equations of neutral type, some very interesting and important problems on the stability of difference equations have arisen. For example, consider the difference equation,

$$x(t) = \sum_{k=1}^{N} A_k x(t-r_k),$$

where each $r_k > 0$ and each A_k is an $n \times n$ matrix. If this equation is stable for one set of values $(r_1,\ldots,r_N)$, is it also stable for values close to these? The answer in general is no. Hale [32] has shown that if stability is preserved, then the equation must be stable for all values of $(r_1,\ldots r_N)$. In his thesis supervised by Hale, Silkowskii [33] has given necessary and sufficient conditions for this type of stability to hold.

Silkowskii [33] has also given a method easier to apply than Pontryagin for obtaining the stability of solutions of linear differential-difference equations with constant coefficients. Tsen, under the direction of Infante, is continuing to work on these important stability questions.

Many theoretical results on fixed points of mappings have arisen because of the discussion of the existence of periodic orbits of periodic dissipative systems (see, for example, Hale and Lopes [34], Chow and Hale [35], Hale [36]). These results were applied by Lopes [37] to equations of neutral type. The results also have implications on uniformly ultimate boundedness and the basic definitions of stability (see the forthcoming book of Hale [38]).

b. Optimal Control of Systems with Delays: Approximation Techniques for Linear, Bilinear, and Weakly Nonlinear Systems

Banks has continued his investigation of approximation methods for optimal control problems governed by autonomous functional

differential equations. During the past year, he has completed a rather extensive study for linear systems of both theoretical and numerical aspects of a method based on use of "averaging" approximations formulated in the context of a framework that is a modification of the one detailed in [39]. The numerical results (which substantiate theoretical findings that this method is indeed a good one for a large class of linear system problems) are reported in [40]. In that report a generous supply of examples (including some involving systems such as those modeling a harmonic oscillator with delayed damping or delayed restoring force) were solved both analytically (using the necessary and sufficient conditions for optimal control of delay systems - developed previously by Banks among others), and numerically (via the "averaging" approximation techniques) and the solutions compared.

A modification (which allows the treatment of a larger class of approximation techniques within the context of the framework) of the conceptual framework in [39] along with new theoretical results for the "averaging" approximation methods were developed in [41] for optimal control problems with (n-vector) system equations

$$\dot{x}(t) = \sum_{i=0}^{\nu} A_i x(t-h_i) + \int_{-r}^{0} D(s)x(t+s)ds + Bu(t), \quad t \in [0,t_1] \qquad (1)$$

where $0 = h_0 < h_1 < \ldots < h_\nu \leq r$. Briefly, this approximation technique involves solving a sequence of control problems governed by the vector ordinary differential equations (which are approximations to (1))

$$\dot{w}^N(t) = A^N w^N(t) + \mathrm{col}(Bu(t),0,\ldots,0)$$

where w^N is a vector in $R^{n(N+1)}$, A^N is the $n(N+1)$ square matrix (taking $\nu = 1$ in (1))

$$A^N \equiv \begin{pmatrix} A_0 & d_1^N & \cdots & \cdots & d_{N-1}^N & A_1+d_N^N \\ \frac{N}{r}I & -\frac{N}{r}I & 0 & \cdots & \cdots & 0 \\ 0 & \frac{N}{r}I & -\frac{N}{r}I & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \frac{N}{r}I & -\frac{N}{r}I \end{pmatrix} .$$

Here I is the $n \times n$ identity matrix and

$$d_j^N \equiv \int_{-\frac{jr}{N}}^{-\frac{(j-i)}{N}} D(s)ds \quad , \qquad j = 1,2,\ldots,N.$$

In [41] there is also given a thorough discussion of the relation of our results to a number of heuristic (and, in some cases, incorrect) uses of similar higher-order ODE approximation ideas for FDE found in the engineering literature during the past 8-10 years. Our analysis has yielded precise convergence results along with error estimates (see [42]). In addition, Banks has recently succeeded in extending some of these approximation ideas to treat certain pro-

blems with nonlinear systems of the form:

$$\dot{x}(t) = L(x_t) + f(x(t), x_t, u(t)), \qquad t \in [0, t_1]. \tag{2}$$

Here L is the same linear operator (on x_t, where $x_t(\theta) = x(t+\theta)$, $-r \leq \theta \leq 0$) as given in the right side of (1) above. Examples of systems which are included in the extended theory for (2) are bilinear control problems of a somewhat standard type arising in applications and nonlinear systems of the type currently under investigation in models for protein synthesis. Details of these results along with a discussion of these models can be found in [42]. Work on extensions of these ideas to other nonlinear problems is continuing.

c. Vector Liapunov Functions and Stability Theory of Ordinary Differential Equations

Following ideas that first appeared in [43] and [44], LaSalle corrected a result in [43] in formalizing an idea due to the economist Arrow for the construction of a Liapunov function from a number of scalar functions, none of which need be a Liapunov function. Arrow did not express his idea in these terms. This can be viewed as a vector Liapunov. LaSalle has further studied the idea of vector Liapunov functions and used them to investigate and obtain new results on global asymptotic stability (see [45]). This more general ide . of a vector Liapunov function, which arose quite naturally in economics, should be useful in deriving certain types of control laws. This has not yet been explored.

The deeper knowledge that we now have of the invariance properties of the limit sets of the solutions of ordinary differential and difference equations in the nonautonomous case increases in importance a type of theorem due originally to Yoshizawa and later modified by LaSalle. The theorem has to do with the set E associated with a Liapunov function. The application of the newer invariance results requires a further improvement in Yoshizawa's result. By extending the concept of a Liapunov function for non-autonomous systems, LaSalle has given a newer version of Yoshizawa's theorem. The conditions imposed are weaker than those of Artstein and also include a recent result given by Onuchic et al in [46]. These results of LaSalle and some new sufficient conditions for asymptotic stability and instability can be found in [47] and [48].

From time to time during the past 5 years LaSalle has thought a great deal about the problem of the stability of feedback structures for the implementation of optimal control without much success. It is clear that in the absence of perturbations there can be an infinity of feedback structures, all of which give the same optimal performance. Which of these is in some sense the "best" or, at least, possess some stability under perturbations? This is the practical problem engineer's solve in building real systems. It should be possible to develop a general theory and to discover some general principles. We have in the past proposed studying this problem. LaSalle has done so but, as was said above, without success. The few simple examples where the problem can be solved are too trivial to be helpful in finding a suitable mathematical formulation of the

general problem. They do, however, show that, even for ordinary differential equations, the available feedback structures (those that are physically realizable) immediately take one beyond ordinary differential equations, and this is the difficulty. One idea here is to study the problem for discrete systems but this would seem to require first a further development of the theory of discrete processes.

d. Difference Equations

Important mathematical models are derived from the observation at discrete times of continuous processes. Most of the data in many real problems of process control is available only at discrete times. LaSalle has noted that, taking a general point of view of a continuous process (general enough to include all the usual mathematical models --ordinary and functional differential equations, etc.) the observed discrete process is equivalent to a system of difference equations on the state space, which may or may not be finite dimensional. The discrete observation of processes generated by ordinary differential equations yield what engineers call "sampled data systems". Difference equations, even in the finite dimensional case, reflect aspects of reality not covered by ordinary differential equations. Not every finite dimensional system of difference equations can be generated by the discrete observation of a system of ordinary differential equations. That this is so is easily seen from the fact that there is in existence and uniqueness of solutions of difference equations only in the forward direction

of time -- two different past histories can lead to the same state but from then on the solution is unique. This is expected whenever there are delayed effects in the dynamics of the system.

Much of what we have learned recently about differential equations and dynamical systems has not been applied to the study of these simple, but practically important, discrete models. For this reason LaSalle began last summer a study of discrete processes and has obtained a number of new results in the theory of difference equations. For example, LaSalle has done for nonautonomous difference equations what Artstein (see the Appendix by Artstein in [49] did for nonautonomous ordinary differential equations in the study of limiting equations and invariance properties. This has enabled him to extend the earlier work of Hurt in [50] in applying the invariance principle to extend Liapunov's direct method. An exposition of some of these results can be found in [49] and [51]. LaSalle is writing, and has partially completed, a book giving a modern treatment of the theory of difference equations (discrete processes) with emphasis on their stability.

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The problem of arbitrarily assigning closed loop poles of a linear multivariable system developed a new method, a generalization of the classical root locus method. Studies have also been conducted on the attainment to stable solutions of model matching problems. A technique was developed, based on a modified minimum energy regulator problem, to obtain feedback stabilization of linear time varying differential systems. Two methods of parameter identification for linear differential systems were developed. A study was made of bilinear control systems with applications to parachute gliding systems and the pursuit-evasion missile. →

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20. Abstract (Continued)

control problem. Studies were made of linear operator feedback for the compensation and control of multivariable systems. A number of computational methods and techniques for control problems with diffusion models were developed in addition to the study of the application of Monte Carlo methods for the optimization of constrained noisy systems. The study of bifurcation problems has been pursued from the abstract viewpoint and for specific applications. Studies were continued for systems described by ordinary and functional differential equations.